University of Turkish Aeronautical Association

PHY 102

MIDTERM EXAMINATION

Saturday, 06 April, 2019

TIME: 10:40-12:30 **110 minutes**

Check student's ID card and write his/her initials here:

Fill in this part completely. Use pen, not pencil:

LAST NAME	FIRST NAME	NO	SECT	DEPT	SIGNATURE

GRADING:

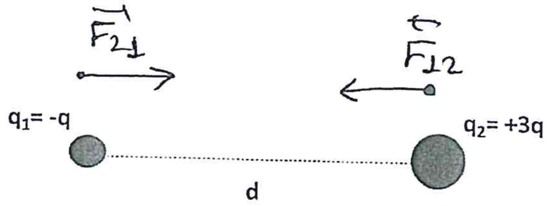
Problem No.	Points	Grader's Name	Grade	Grade Change (If any after objection.)
1	20	EDT		
2	20	SD		
3	20	MAO		
4	20	SB		
5	20	ÇŞ		
Total =	100	Total =		

Solutions

- This examination has 5 classical type problems. Solve all of them.
- Check the exam. If there are any missing pages, missing problems, or printing errors, inform the proctor immediately.
- **Use of calculator and asking questions to the proctor is not allowed.**
- If you think that a problem is wrong, do not spend time. Skip it and continue, and see the coordinator after the exam.
- There might be corrections during the exam. Listen carefully and do the corrections on the exam paper.
- If a problem has quantities given like $x, y, V, E, r, q, \ldots$ etc. with no numerical values, then express your answers in terms of some or all of the given quantities and the related constants like the permittivity of empty space ε_0, etc.
- **Write all steps in answering a problem in the space provided under each problem. If the solution steps are not shown, no credits will be given. Write clearly all formulas you use. SI units must be given for all numerical answers.**
- **Students cannot leave the exam room during the first 15 minutes. Any student who comes late more than 15 minutes cannot take the exam. No extra time is given to the student who comes late during the first 15 minutes.**
- **Keep your solutions very carefully away from other eyes around you by keeping the exam papers folded at all times. Otherwise another student may look at your solution and copy it. If that happens, then you will also be questioned.**

Instructors: Assoc. Prof. Dr. S. BADOĞLU, Assoc. Prof. Dr. S. DENGİZ, Dr. M. A. OLPAK, Assoc. Prof. Dr. Ç. ŞENTÜRK, Assoc. Prof. Dr. E. D. TEKIN

Q1. Two unequal point charges, one with charge $q_1 = -q$ and the other with charge $q_2 = +3q$, are separated by a distance **d**.



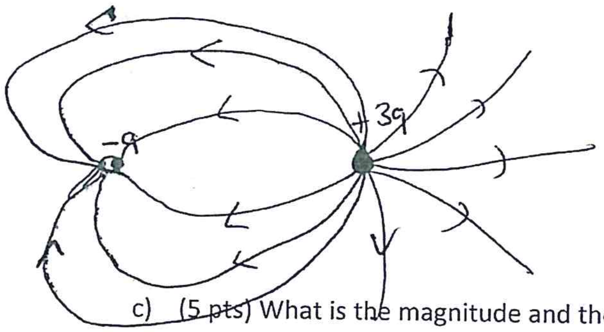
a) (5pts) Write the magnitudes and show the directions of the force that q_1 exerts on q_2 ($\vec{F}_{12}$) and the force that q_2 exerts on q_1 ($\vec{F}_{21}$).

$$F_{21} = k\frac{(+3q)(-q)}{d^2} = \boxed{-\frac{k3q^2}{d^2}}$$

$$F_{12} = \frac{k.(-q)(3q)}{d^2} = \boxed{-\frac{k3q}{d^2}}$$

They are same!
(Also; Newton's 3rd law $F_{12} = F_{21}$)

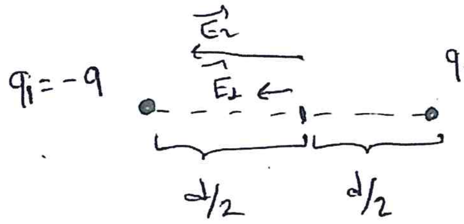
b) (5pts) Draw below the electric field lines between these two charges. Your drawing should be as accurate as possible. Explain your drawing.



→ Number of lines is proportional to the magnitude of charge!

→ Thus; three times more lines leave +3q as there are lines entering "-q"!

c) (5 pts) What is the magnitude and the direction of the electric field at a point halfway between the charges?

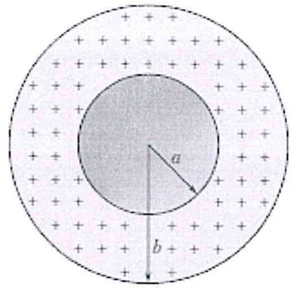


$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$= k.\frac{(q)}{(d/2)^2} + k.\frac{(3q)}{(d/2)^2}$$
$$= \frac{4kq}{d^2} + \frac{12kq}{d^2}$$
$$\vec{E} = \frac{16kq}{d^2} \quad \text{(to the left)}$$

d) (5pts) What is the work to be done to increase the distance between particles from **d** to **2d**?

$$W_{ext} = \Delta U = U_2 - U_1$$
$$= \left(-k.\frac{3q^2}{2d}\right) - \left(-\frac{k3q^2}{d}\right)$$
$$= -k.\frac{3q^2}{2d} + k.\frac{6q^2}{d} \Rightarrow \boxed{W_{ext} = \frac{3kq^2}{d}}$$

PROBLEM 2 (20 points)



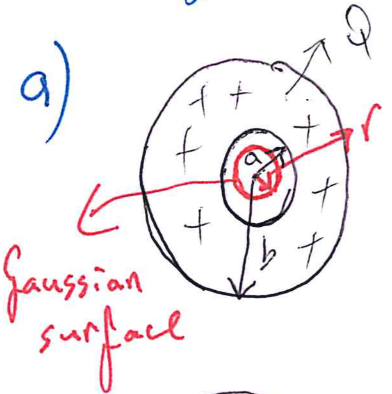
Suppose the *nonconducting* sphere has a spherical cavity of radius a and centered at the sphere's center. By assuming the charge Q is distributed uniformly in the "shell" (that is, between $r = a$ and $r = b$), determine the electric field as a function of r for

a) $0 < r < a$, b) $a < r < b$, c) $r > b$

(Hint: Use Gauss' Law.)

The system is spherical symmetric. So, we use Gauss' Law for the sake of simplicit

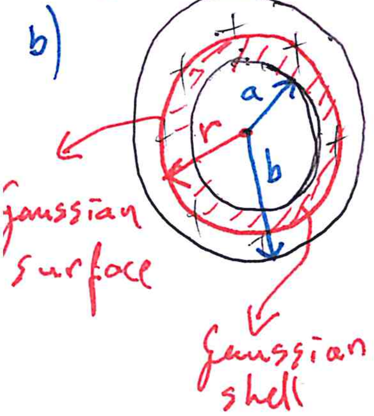
a)



→ Note that for $0<r<a$, $Q_{enc} = 0$.

→ So, from Gauss' Law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \boxed{E_{r<a} = 0}$

b)



• To apply Gauss' Law, let us first find Q_{enc}:

→ For this purpose, let us recall that the volume charge density is:

$$\rho = \frac{Q}{V_{shell}} = \frac{Q}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3} = \frac{Q}{\frac{4\pi}{3}(b^3 - a^3)}$$

Hence,

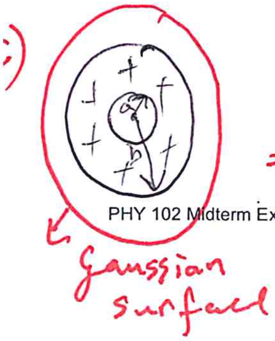
$$Q_{enc} = \rho V_{\substack{Gaussian\\shell}} = \frac{Q}{\cancel{\frac{4\pi}{3}}(b^3-a^3)}\left(\cancel{\frac{4}{3}\pi} r^3 - \cancel{\frac{4}{3}\pi} a^3\right)$$

$$\Rightarrow Q_{enc} = \frac{r^3 - a^3}{b^3 - a^3} Q$$

Insert Q_{enc} into the Gauss' Law:

$$\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{1}{\epsilon_0}\left(\frac{r^3-a^3}{b^3-a^3}\right)Q \Rightarrow \boxed{E_{a<r<b} = \frac{Q}{4\pi\epsilon_0 r^2}\left(\frac{r^3-a^3}{b^3-a^3}\right)}$$

c)

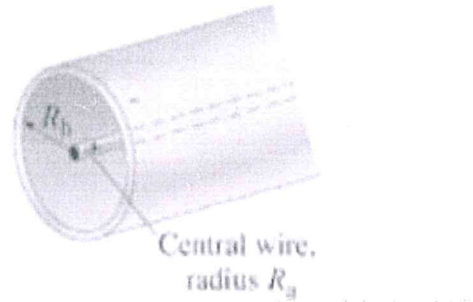


For $r>b$, $Q_{enc} = Q$

$$\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E_{r>b} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}}$$

PROBLEM 3 (20 points)

a) (10 pts.) Consider a system of two infinitely long conductors, as shown in the figure below. One of the conductors is a wire of finite thickness, with radius R_a. The other is a cylindrical shell with radius R_b, whose axis concides with the wire. The wire carries charge per unit length λ (positive, in units of C/m), and the shell carries charge per unit length $-\lambda$. Show that the potential difference between the wire and the shell is $V = \frac{\lambda}{2\pi\epsilon_0} ln\left(\frac{R_b}{R_a}\right)$.



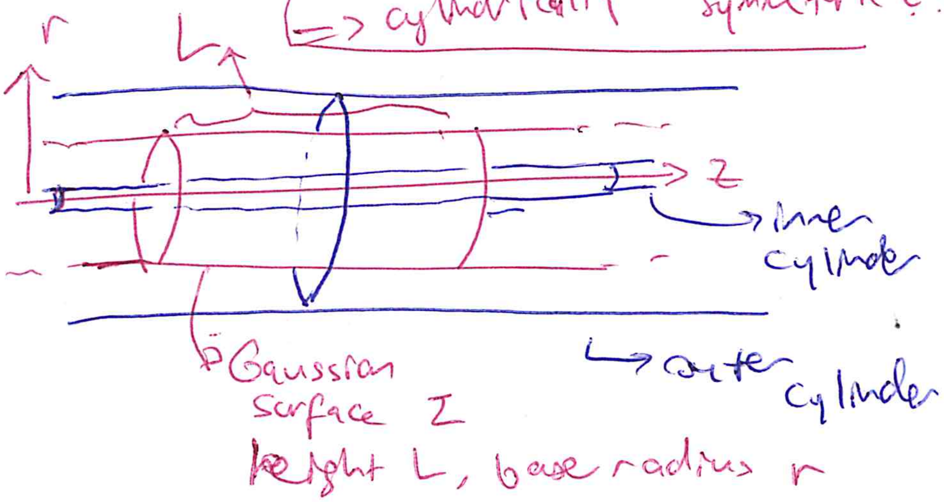
b) (5 pts.) Four equal point charges, each having value Q, are fixed at the corners of a square of side L. What is the total electrostatic potential energy of the system? Draw a figure and express the given quantities on your figure.

c) (5 pts.) The electric potential in a region of space is given as $V = \frac{by}{a^2+y^2}$, where a, b are constants and y is the Cartesian y-coordinate. Calculate the corresponding electric field within the same region of space.

a) To find the potential difference between the conductors, we need the electric field in the region $a < r < b$.

Long cylinders & constant charge density
coaxial
$\Rightarrow$ cylindrically symmetric $\vec{E}$:

$\vec{E} = E_r(r)\hat{r}$ where



$$\oint_\Sigma \vec{E}\cdot d\vec{a} = \frac{Q_{enc.\ by\ \Sigma}}{\epsilon_0}$$

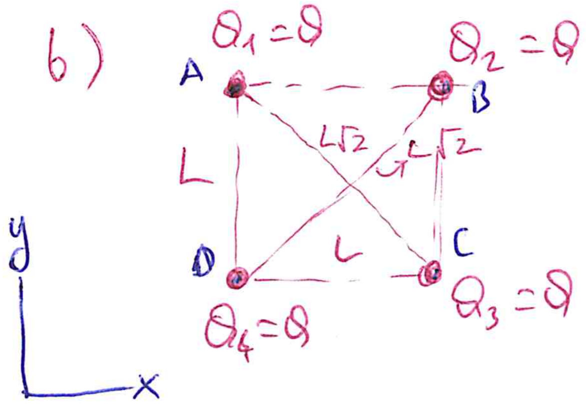
$$E_r \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E_r = \frac{\lambda}{2\pi\epsilon_0 r} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

$$\Rightarrow V(R_a) - V(R_b) = -\int_{R_b}^{R_a} \vec{E}\cdot d\vec{l} = -\int_{R_b}^{R_a} \frac{\lambda}{2\pi\epsilon_0 r}dr$$

$$\boxed{V(R_a) - V(R_b) = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{R_b}{R_a}\right)}$$

b)



Q_1 is sitting at A
Q_2 " " " B
Q_3 " " " C
Q_4 " " " D

$|AB| = |BC| = |CD| = |DA| = L$

$|AC| = |BD| = L\sqrt{2}$

$$U = \frac{Q_1 Q_2}{4\pi\epsilon_0 |AB|} + \frac{Q_2 Q_3}{4\pi\epsilon_0 |BC|} + \frac{Q_3 Q_4}{4\pi\epsilon_0 |CD|} + \frac{Q_4 Q_1}{4\pi\epsilon_0 |DA|}$$

$$+ \frac{Q_1 Q_3}{4\pi\epsilon_0 |AC|} + \frac{Q_2 Q_4}{4\pi\epsilon_0 |BD|}$$

$$= \frac{1}{4\pi\epsilon_0} \left\{ \frac{Q^2}{L} + \frac{Q^2}{L} + \frac{Q^2}{L} + \frac{Q^2}{L} + \frac{Q^2}{L\sqrt{2}} + \frac{Q^2}{L\sqrt{2}} \right\}$$

$$= \frac{Q^2}{4\pi\epsilon_0 L} \left(4 + \frac{2}{\sqrt{2}} \right) = \boxed{\frac{Q^2}{4\pi\epsilon_0 L} * (4 + \sqrt{2}) = U}$$

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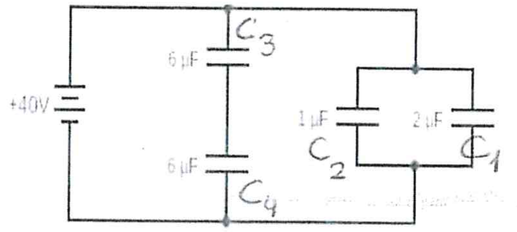
c) $V = \dfrac{by}{a^2 + y^2} = V(y) \Rightarrow \vec{E} = -\dfrac{dV}{dy} \hat{j}$

$$= -\frac{d}{dy} \left(\frac{by}{a^2+y^2} \right) \hat{j}$$

$$= -\frac{b(a^2+y^2) - 2y \cdot by}{(a^2+y^2)^2} \hat{j}$$

$$\boxed{\vec{E} = \frac{b(y^2 - a^2)}{(a^2+y^2)^2} \hat{j}}$$

PROBLEM 4 (20 points)



Evaluate the circuit shown below to determine the equivalent capacitance, and then the charge and voltage across each capacitor.

$$C_{12} = C_1 + C_2 = 2\,\mu F + 1\,\mu F = 3\,\mu F$$

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6\,\mu F} + \frac{1}{6\,\mu F} = \frac{1}{3\,\mu F}$$

$$\Rightarrow C_{34} = 3\,\mu F$$

$$C_{eq} = C_{12} + C_{34} = 3\,\mu F + 3\,\mu F = 6\,\mu F$$

Since they are parallel branches: $V = 40\,V = V_{34} = V_{12} = V_1 = V_2$

Hence; $V_1 = V_2 = 40\,V$

Then; $Q_1 = C_1 V_1 = (2\,\mu F)(40\,V) = 80\,\mu C$

$Q_2 = C_2 V_2 = (1\,\mu F)(40\,V) = 40\,\mu C$

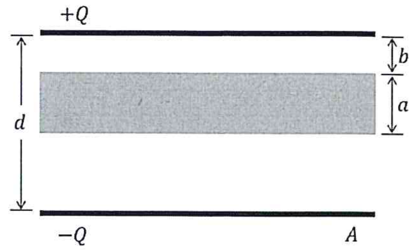
And $Q_{34} = C_{34} V_{34} = (3\,\mu F)(40\,V) = 120\,\mu C$

Since C_3 and C_4 are in series, $Q_3 = Q_4 = Q_{34} = 120\,\mu C$

Then; $V_3 = \frac{Q_3}{C_3} = \frac{120\,\mu C}{6\,\mu F} = 20\,V$

$V_4 = \frac{Q_4}{C_4} = \frac{120\,\mu C}{6\,\mu F} = 20\,V$

PROBLEM 5 (20 points)



A parallel-plate capacitor has a plate separation d and plate area A. After the capacitor is charged and then the battery is disconnected, an uncharged conductor of thickness a is placed between the plates of the capacitor, as shown in the figure. The electric field between the plates when the conductor is not present has the magnitude $E_0 = \frac{Q}{\epsilon_0 A}$.

a) (8 pts.) Show that the capacitance of this arrangement is $C = \epsilon_0 \frac{A}{d-a}$.

b) (5 pts.) Find the change in the capacitance of the capacitor after this process. Is the capacitance increased or decreased by the insertion of the conductor.

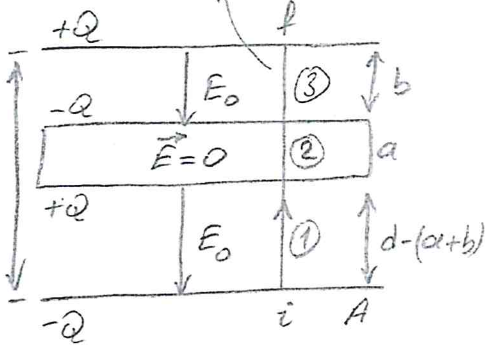
c) (7 pts.) What is the work required to remove the conductor from the system completely. (Since the battery is disconnected, the charge remains constant during this process!)

Sol.

(Path of integration)

(Pot. diff. across the plates)

a) (8)



$$V_{fi} = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{l}$$

$$= -\int_{①} \vec{E}_0\cdot d\vec{l} - \int_{②} \vec{E}\cdot d\vec{l} - \int_{③} \vec{E}_0\cdot d\vec{l}$$

(In equilibrium, $\vec{E}=0$ within the conductor)

$$= E_0(d-a-b) + E_0 b$$

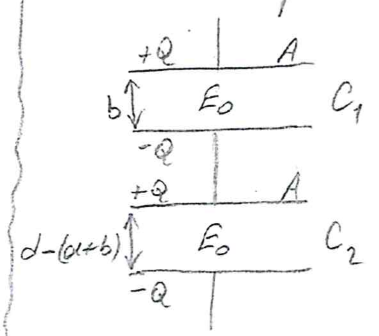
$$= E_0(d-a) > 0$$

$[\vec{E}_0\cdot d\vec{l} = E_0\, dl \cos 180 = -E_0\, dl]$

$$\Rightarrow C = \frac{Q}{|V_{fi}|} = \frac{Q}{E_0(d-a)} = \epsilon_0 \frac{A}{d-a} \quad //$$

$$\left[E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}\right]$$

II. way: The system is equivalent to two capacitors in series:



$$C_1 = \epsilon_0 \frac{A}{b} \quad \& \quad C_2 = \epsilon_0 \frac{A}{d-a-b}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d-a}{\epsilon_0 A}$$

$$C = \epsilon_0 \frac{A}{d-a}$$

$\Longrightarrow$

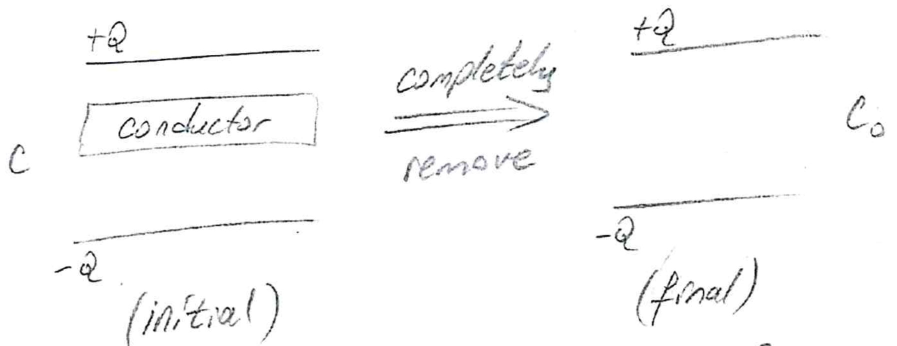
b) $C_0 = \epsilon_0 \frac{A}{d}$ $\Rightarrow$ capacitance in the absence of the conductor
(5)

$C = \epsilon_0 \frac{A}{d-a}$ $\Rightarrow$ capacitance with the conductor

$$\Delta C = C - C_0 = \epsilon_0 \frac{A}{d}\left(\frac{d}{d-a} - 1\right) = \frac{a}{d-a} C_0 > 0 \quad //$$

$\Rightarrow$ Since positive, C increases!

c) Since charge remains const. during the removal
(7) process,



$U_i = \frac{1}{2}\frac{Q^2}{C}$ $\qquad$ $U_f = \frac{1}{2}\frac{Q}{C_0}$

$$\Rightarrow W = \Delta U = U_f - U_i = \frac{Q^2}{2}\left(\frac{1}{C_0} - \frac{1}{C}\right)$$

$$= \frac{Q^2}{2}\left(\frac{d}{\epsilon_0 A} - \frac{d-a}{\epsilon_0 A}\right) = \frac{Q^2 a}{2\epsilon_0 A} \quad //$$