GN-4. EXPERIMENTAL ERRORS

Physics is based *on measurement.* We must learn how to measure the *physical quantities* in terms of which the laws of physics are stated. *Length, time, mass, speed, velocity, acceleration, force, momentum, temperature, charge, voltage, current, resistance* ...... are all physical quantities. We use some of these words in our everyday speech very frequently. In physics they have precise meanings, sometimes different than their everyday meanings.

In the physics laboratory, and in any other laboratory, we are going to perform experiments. Although the aim of these experiments is very different, they all commonly involve *measuring and recording quantities,* and deriving from these recorded measurements other quantities which we are interested in determining. We may be measuring the distance between two points on the track of a moving object and the time interval that has passed to go between these points. We may be measuring the electric current flowing through a resistance wire and the voltage across it when the resistance wire is connected across a battery in an electrical circuit.

Can a simple quantity like the distance between two points on a track be measured exactly? To measure this distance we use a ruler by laying it along the track and make observations at the two points. We first try to set one of the points at zero on the ruler, but how exactly can we set it at zero?

As we know the zero mark on the ruler is usually a line and the line has a certain thickness, which prevents us from getting an exact setting at zero. The same is true for the other point. Reading a value from a ruler involves lining up the two points with the marks on the ruler, and as you can see easily, exact lining up is impossible. The apparent distance between two points, and therefore the values of the reading, depends also on the position of your eye. For example, the position of a point near a ruler may appear different if viewed from the left or right of a line of sight perpendicular to the ruler, as shown in Fig.4-1.

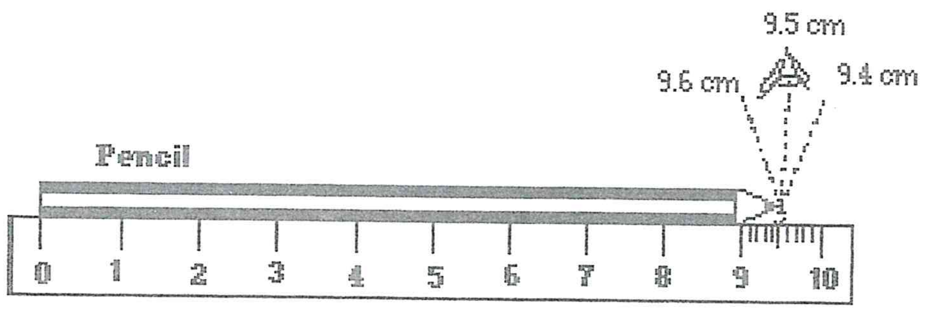

FIGURE 4-1 Example of parallax in reading the position of a point from a ruler.

A reading may appear to be different when viewed with our left eye or right eye, or when we move our head horizontally or vertically over a ruler. This apparent change in position due to a change in the position of the eye is called *parallax.* There is another possibility that the ruler we use may not be accurate. The size of the ruler may vary with such factors as time, temperature, or humidity.

The scale of a measuring instrument is not exact. The exact reading of a scale is impossible. The exact measurement of any quantity is impossible. No experiment gives the right answer.

In the physics laboratory the measurements are carried out by means of instruments, such as a ruler, a voltmeter or an ammeter, and whether we like it or not, in all measurements there is always some *error* (or *uncertainty).* We aim to get nearer to the right value using more accurate instruments and making our measurements more carefully.

Sometimes, we try to make a rough estimate of a quantity and, in such cases, the accuracy of the instrument we use and the way we make our measurement are not very important. In most cases, however, we want to be as accurate as possible. But more than that, we always want to know how accurate our measurements are and how accurate the result which we obtain from our measurements is.

When we want to know how accurate a measurement is, we refer to what is called the *error in this measurement.* The word error does not have the meaning mistake, but rather it is the uncertainty in the measurement as a result of all the contributing factors.

As an experimenter, we have to make a reasonable assessment of the accuracy of our measurements detecting the sources of errors and we have to estimate the maximum possible error in each of our measurements.

Accuracy expresses how closely the measurement comes to the true or known value. **Precision** or **reproducibility** expresses the deviation of a measurement from the average of many measurements using the same procedure repeatedly.

RECORDING MEASUREMENTS

In recording a measurement, it is important to *estimate and record the error.* For example, if we measure the distance between two points on a track with a ruler having millimeter division on it, we may record our measurement as

$$Distance = 2.5 \pm 0.1\ cm$$

The significance of ± 0.1 cm is that when we repeat our measurement several more times, the readings we obtain are expected to be one of the values 2.4 cm, 2.5 cm, or 2.6 cm. For many repetitions of the measurement, the value most frequently occurring will be the value 2.5 cm. Note that all readings require both *a number* and *a unit.*

The **error in a measurement** can be either positive (reading too much) or negative (reading too little).

HOW ERROR IN A MEASUREMENT IS INDICATED

We can indicate the error in a measurement in the following two ways:

a) We follow the numerical result of our measurement by the symbol ± and then the maximum possible error,

b) We write down the numerical result of our measurement giving only the figures read from the scale. The last figure given is generally the one in which there is some uncertainty or error.

Look at the following table, which summarizes the experimental values obtained for the speed of light. All the methods used a rotating mirror set up for the measurement. As we see from Table 4-1, no experiment gives the true value of a physical quantity. The experimenter tries to get close to the true value, as Michelson did for the measurement of the speed of light. Notice that he initially could make measurements with an error of ± 50 km/s, finally he could reduce the error down to ± 4 km/s. The accepted value for the speed of light today is

$$299\ 792\ 459.0 \pm 0.8\ m/s$$

which is in accordance with Michelson's value in 1926, *within the estimated limits of error*.

Date	Experimenter	Speed of Light (km/s)
1875	Cornu	299 990 ± 200
1880	Michelson	299 910 ± 50
1883	Newcomb	299 860 ± 30
1883	Michelson	299 850 ± 60
1926	Michelson	299 796 ± 4

TABLE 4-1. Determination of the speed of light using the rotating mirror method

TYPES OF ERRORS

In general, the types of errors in a particular experiment may be due to the observer, the measuring instrument used, or the experimental design used. All of the possible errors can be considered in two main groups, *systematic* and *random* errors.

SYSTEMATIC ERRORS:

They are the errors tending to be in one direction only, either positive or negative. They will produce a result, which is always wrong in the same way. If a voltmeter reads 1.2 V when the true voltage is 1.5 V, then there is a systematic error of –0.3 V. The systematic error is not necessarily constant over the entire scale of a measuring instrument. The voltmeter we are considering, when the true voltage is zero, will probably read zero and there is no systematic error.

The usual source of systematic errors is the construction or calibration of the measuring instrument. For example, an instrument whose zero setting is wrong gives rise to systematic error and it systematically gives an incorrect reading, either larger or smaller, instead of a true value. We can calibrate an instrument using *a standard* whose value is known accurately.

Systematic errors due to improper calibration, zero setting, etc., can be avoided by proper inspection and adjustment of the instruments and their elimination depends on the skill of the experimenter. In carrying out an experiment it is obviously important to consider possible sources of systematic errors and to take precautions to eliminate them as much as possible.

RANDOM ERRORS:

They are the errors tending to be in both directions, negative and positive. These errors are random and produce results, which are both too large or too small. These errors arise from unknown and unpredictable variations in the experimental situation. For example, variation due to parallax or human judgment in interpolating between marks on a ruler used to measure distance between two points produce random error in the result of measurement. They are beyond the control of the experimenter. Inaccurate reading of a scale, unpredictable fluctuations in temperature or line voltage, mechanical vibration of the experimental set up, carelessness in making proper measurement, and other similar
accidental reasons can give rise to random errors. They are positive as often as they are negative. We can reduce the effect of random errors by improving the experimental technique, by greater care, by experience and practice, and by improved instruments etc.

One way to reduce the random error in a measured quantity is to take the measurement many times and calculate the average of these independent measurements. This will likely be more accurate than a single measurement.

ESTIMATING ERRORS

Accepting that any measurement is to be in error, it is very important to be able to make good estimates of the maximum possible error or uncertainty in each recorded measurement.

The result of a measurement conveys knowledge about a physical quantity and it is a scientific statement. The statement is wrong if it says less or more than is known. The most accurate statement is the one, which says just what is known and no more.

When reading a ruler, which is marked in millimeter, we should be able to read to the nearest division. Our maximum reading error should then be ± 1 mm. We may attempt to estimate our reading to less than one division, for example a half or a fifth of a division, by making our reading more carefully. The error in these cases would be ± 0.5 mm or ± 0.2 mm, respectively. Certainly, ± 0.2 mm is an overestimate since with the naked eye a fifth of a millimeter division is very difficult to observe. A half of a millimeter, ± 0.5 mm can be observed easily and it is better always to be on the cautious side. For example, a reading from such a ruler might be recorded as 25.5 ± 0.5 mm or 2.55 ± 0.05 cm. Notice that, the last figure shown is the one in which there is some uncertainty.

GN-5. SIGNIFICANT FIGURES AND RECORDING MEASUREMENTS

As mentioned previously, one of the ways of indicating the error in a measurement is by the number of figures recorded. This requires that all the figures recorded (including the last one estimated) are significant. For example, for the distance measured with a ruler marked in millimeters we recorded 2.55 ± 0.05 cm, so this distance would be written with three significant figures, 2.55. If we record our measurement (like 2.55 cm) using only significant figures, the amount of error in the measurement is not specified. The measurements 2.55 ± 0.01 cm, 2.55 ± 0.02 cm or 2.55 ± 0.05 cm would all have three significant figures and would all be recorded as 2.55 cm.

In recording measurements, the correct number of significant figures must be used and when possible, the more definite indication $\pm$ must be added. When we use the significant figure method to record measurements, we need to develop rules for expressing the number of significant figures in calculated results.

Give the figures you read from a scale with only the last figure estimated as a result of measurement. Never put extra figures. The digits including the last one estimated are called **significant figures**.

The number of significant figures has nothing to do with the location of the decimal point. Care should always be taken to distinguish between zeros that are significant and those that are not. Thus, if in measuring the distance with a ruler marked in millimeters, the position of the point appeared to be right opposite the 5 mm division following the 2 cm mark, it would be recorded as 2.50 cm and the zero would be significant. However, if the distance measurement were to be stated in meters, like 0.0250 m, or in micrometers (1 micrometer = 1 μm = 1×10^{-6} m); like 25500 μm, the zeros other than the one which is to the right of 5 would not be significant and they would be serving only to place the decimal point. The number of significant figures in a measurement is determined as follows:

1. The leftmost nonzero digit is the most significant.

2. If there is no decimal point, the rightmost nonzero digit is the least significant.

3. If there is a decimal point, the rightmost digit is the least significant, even if it is zero.

4. All digits between the least and most significant digits are considered to be significant.

Example 5-1

Some measurements are recorded as: 255 mm; 11.1 cm; 2.04 V; 175000 mA; 0.0125 km; 360 g. How many significant figures do we have in these measurements?

Solution:

We have three significant figures in each of these measurements.

A difficulty usually arises if the decimal point is omitted and the rightmost digit is zero. For example, the measurement 3.60 m has three significant figures, but what about if the measurement were recorded as 360 cm? By the rules given above, the result 360 cm has only two significant figures, but the last digit zero is actually significant. Without knowledge that the original measurement of 3.60 m contained three significant figures, however, there is no way to tell whether the zero in 360 cm is significant or not.

This problem is resolved by writing the measurements *in scientific notation or powers-of 10 notation.* Thus, writing 360 cm as 3.60×10^2 cm shows explicitly that the rightmost zero is significant.

> ***Scientific notation:*** Write the result of a measurement as two factors; the first factor contains all the significant figures, having one nonzero digit in front of the decimal point, and the second factor is a power-of-10.

Let us write the measurements in Example 5-1 in scientific notation:

$$255\ mm = 2.55 \times 10^2\ mm$$
$$11.1\ cm = 1.11 \times 10^1\ cm$$
$$2.04\ V = 2.04 \times 10^0\ V$$
$$175\ 000\ mA = 1.75 \times 10^5\ mA$$
$$0.0125\ km = 1.25 \times 10^{-2}\ km$$
$$360\ g = 3.60 \times 10^2\ g$$

Note that in each case above the first factor gives the number of significant figures existing in the corresponding measurement.

CALCULATION WITH SIGNIFICANT FIGURES

The calculations carried out with significant figures usually produce extra figures as a result of calculation. However, reporting more figures would imply greater significance than that given by the measurements and a result can not be made more accurate by a calculation. As we know, the last figure of a measurement is estimated and therefore it is doubtful. We may expect the result of calculation to have also only one doubtful figure. This means that only the first doubtful figure from the left of a result is to be reported.

When a doubtful figure is added, subtracted, multiplied or divided by another doubtful (or significant) figure, the resultant figure is also doubtful. To show how doubtful figures are carried through a multiplication operation, the estimated (doubtful) figures are indicated by boldface numbers as in the following example:

$$\begin{array}{r} 1.23\mathbf{1} \\ 1.\mathbf{5}\ \text{(least accurate)} \\ \times \overline{\qquad\quad} \\ \mathbf{5755} \\ 123\mathbf{1} \\ + \overline{\qquad\quad} \\ 1.\mathbf{8065}\ (\cong 1.\mathbf{8}) \end{array}$$

The first doubtful figure from the left is **8** and therefore the result of this multiplication is rounded off to 1.**8**. Note that the result has two significant figures, 1 and 8, as the least accurate number 1.**5** used in the calculation.

Multiplication and Division: In the multiplication or division of numerical measurements, retain in the result only as many figures as the number of significant figures in the least accurate number used in the calculation.

To show how significant figures are carried through an addition operation, again indicating the doubtful figure by boldface letters, let's look at the following example:

$$\begin{array}{r} 44.214\mathbf{6} \\ 0.1\mathbf{6} \\ 120.\mathbf{4} \\ 1.12\mathbf{2} \\ + \overline{\qquad\qquad} \\ 165.\mathbf{8966}\ (\cong 165.\mathbf{9}) \end{array}$$

Note that, the **4** in the 120.**4** is the first doubtful digit from the left and it defines the position of the figure where doubt occurs. The first doubtful figure from the left is **8** and the result of this addition is rounded off to 165.**9** since the figure after **8** is **9**. The rules of rounding off numbers are given in the following paragraph. Note also that, in addition (or subtraction) we can not use the actual number of significant figures in the numerical measurements, but we pay attention to their positions.

Addition and Subtraction: When carrying out addition or subtraction with numerical measurements, do not carry the result beyond the first column (from the left) that contains a doubtful digit.

The numerical measurements can be *rounded off* to the desired number of significant figures before they are used in calculations. A numerical measurement is rounded off according to the following rules:

1. In rounding off, the last figure obtained should be unchanged if the figure to the right of it is less than 5 (0,1,2,3 or 4).

2. The last figure obtained should be increased by 1 if the figure to the right of it is 5 or greater (5,6,7,8 or 9).

According to these rules the number 23.47 is rounded off to three digits as 23.5 and to two digits as 23. Similarly, the number 23.84 is rounded off to three digits as 23.8 and to two digits as 24.

Rounding off the numbers, in the first example above, before we make the calculations, we find

1.231	1.2 (rounded off)
1.5 (least accurate)	1.5
×	×
5755	60
1231	12
+	+
1.8065 ($\cong$1.8)	1.80 ($\cong$1.8)

Similarly, in the second example above, we find

44.2146	44.2
0.16	0.2
120.4	120.4
1.122	1.1
+	+
165.8966 ($\cong$165.9)	165.9

As seen from the results, rounding off the number(s) before the calculation or rounding off the result at the end yields the same value.

Example 5-2:

A student measures three quantities, in appropriate units, and records them using the method of significant figures as: A =12.50; B = 2.72; C =1.4.

a) How many significant figures are there in these measurements?
b) Write these measurements in scientific notion.
c) Round off A and B to two significant figures.
d) What is the result of multiplying $A \times C$?
e) What is the sum of the measurements?

Solution:

a) A =12.50; four significant figures.
B = 2.72; three significant figures.
C =1.4; two significant figures.

b) In scientific notation, we write these measurements
$A = 1.250 \times 10^1$
$B = 2.72 \times 10^0$
$C = 1.4 \times 10^0$

c) Rounding off to two significant figures, we find

$$A = 12.50 = 13$$
$$B = 2.72 = 2.7$$

d) Multiplying by using boldface numbers for doubtful figures, we obtain

$$\begin{array}{r} 12.5\mathbf{0} \\ 1.\mathbf{4} \\ \times\overline{} \\ \mathbf{5000} \\ 125\mathbf{0} \\ +\overline{} \\ \mathbf{17.500}\ (\cong 18) \end{array} \quad \text{or} \quad \begin{array}{r} 1\mathbf{3} \\ 1.\mathbf{4} \\ \times\overline{} \\ \mathbf{52} \\ 1\mathbf{3} \\ +\overline{} \\ 1\mathbf{8.2}\ (\cong 18) \end{array}$$

e) Summing the numbers

$$\begin{array}{r} 12.5\mathbf{0} \\ 2.7\mathbf{2} \\ 1.\mathbf{4} \\ +\overline{} \\ 16.\mathbf{62}\ (\cong 16.6) \end{array} \quad \text{or} \quad \begin{array}{r} 12.\mathbf{5} \\ 2.\mathbf{7} \\ 1.\mathbf{4} \\ +\overline{} \\ 16.\mathbf{6} \end{array}$$

Example 5-3:

Consider the scales of two different voltmeters, one of which is finely divided while the other one is coarse as shown in Fig.5-1. They are properly zeroed and calibrated using *a standard cell* (one whose voltage is known to be accurate) in order to avoid systematic errors. Then, the two voltmeters are connected to the same battery and their scales are shown in Fig.5-1. Read the two scales and record your measurements estimating their errors.

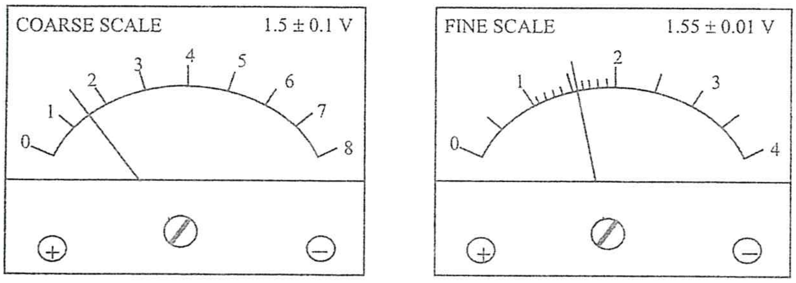

FIGURE 5-1. Two voltmeter scales measuring the same voltage.

Solution:

From the coarse scale we read 1.5 V. Figure 1 in this reading is certain, but figure 5 is estimated. Maximum possible reading error for the coarse scale is estimated as ± 0.1 V and the result of this measurement can be recorded as 1.5 ± 0.1 V. This means that the true value is between 1.4 V and 1.6 V. From the fine scale we read 1.55 V. In this

GN-6. COMBINATION OF ERRORS IN CALCULATED RESULTS

Whenever measurements are used in a calculation, an error is associated with the result obtained. The manner in which the error of each measurement propagates and combines depends upon the mathematical calculation used.

Suppose we have two measurements x and y with respective errors (maximum possible, probable, average or as standard deviation) Δx and Δy. We record our measurements as $x \pm \Delta x$ and $y \pm \Delta y$. The examples that follow discuss the types of calculation we may have with these measurements.

ADDITION AND SUBTRACTION

Suppose we add the measurements $x \pm \Delta x$ and $y \pm \Delta y$ to find the result R and we want to determine the maximum possible error ΔR in R. The calculated value is

$$R = x + y$$

When the errors Δx and Δy happen to reinforce each other, the result R could be as large as

$$\begin{aligned} R + \Delta R &= (x + \Delta x) + (y + \Delta y) \\ &= (x + y) + (\Delta x + \Delta y) \end{aligned}$$

The result R could be as small as

$$\begin{aligned} R - \Delta R &= (x - \Delta x) + (y - \Delta y) \\ &= (x + y) - (\Delta x + \Delta y) \end{aligned}$$

So, we can write the result R with its maximum possible error ΔR as

$$R \pm \Delta R = (x + y) \pm (\Delta x + \Delta y)$$

Thus the absolute maximum possible error is

$$\Delta R = \Delta x + \Delta y$$

When we want to find the difference $R = x - y$, the difference could be as large as

$$R + \Delta R = (x + \Delta x) - (y - \Delta y) = (x - y) + (\Delta x + \Delta y)$$

The difference R could be as small as

$$R - \Delta R = (x - \Delta x) - (y + \Delta y)$$

$$= (x-y) - (\Delta x + \Delta y)$$

So we can write for the difference

$$R \pm \Delta R = (x-y) \pm (\Delta x + \Delta y)$$

The absolute maximum possible error is

$$\Delta R = \Delta x + \Delta y$$

Note that the maximum possible error in both addition and subtraction is the sum of the individual errors in the quantities.

> When measured quantities are **added** or **subtracted**, the maximum possible error in the result is the sum of the errors in the quantities used in the calculation.

When we add two measured quantities $x \pm \Delta x$ and $y \pm \Delta y$, the sum is $(x + y) \pm (\Delta x + \Delta y)$. In writing the sum in this way we assume that the errors Δx and Δy are in the same direction to reinforce each other. However, statistically, the probability of the two errors being in the same direction is equal to the probability of the two errors being in the opposite directions. We would like, of course, them to be in opposite directions and therefore they would cancel out each other, but this can not be counted on. In order to be safe, we assume the worst and use the *maximum possible error* of the result.

Statistically, however, the individual errors Δx and Δy in the measured quantities are most probably combined in such a way that the probable error in the result is the square root of the sum of the squares of the individual errors. Thus,

$$\textit{The probable error in the result} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

For the experiments covered in this manual, the maximum possible error, which is easier to find than the probable error, will mostly be used.

Example 6-1:

A student obtained the distances by making measurements such as

$$x \pm \Delta x = 12.42 \pm 0.01\ cm$$
$$y \pm \Delta y = 8.61 \pm 0.01\ cm$$
$$z \pm \Delta z = 18.36 \pm 0.02\ cm$$
$$t \pm \Delta t = 12.04 \pm 0.02\ cm$$

Calculate the difference $D = x - y$ and the sum $S = z + t$ with their maximum possible errors ΔD and ΔS, respectively.

Solution:

$$\begin{array}{r} 12.42 \pm 0.01 \text{ cm} \\ 8.61 \pm 0.01 \text{ cm} \\ + \overline{\phantom{12.42 \pm 0.01 \text{ cm}}} \\ D \pm \Delta D = 3.81 \pm 0.02 \text{ cm} \end{array} \qquad \begin{array}{r} 18.36 \pm 0.02 \text{ cm} \\ 12.04 \pm 0.02 \text{ cm} \\ + \overline{\phantom{18.36 \pm 0.02 \text{ cm}}} \\ S \pm \Delta S = 30.40 \pm 0.04 \text{ cm} \end{array}$$

Note that the error in each of the calculated results is the sum of the individual errors in both cases. We can check this by using the values, which will give the greatest and smallest results.

Greatest $x = 12.42 + 0.01$ $= 12.43$ cm	Smallest $x = 12.41 - 0.01$ $= 12.41$ cm
Smallest $y = 8.61 - 0.01$ $= 8.60$ cm	Greatest $y = 8.61 + 0.01$ $= 8.62$ cm
Greatest $D = 12.43 - 8.60$ $= 3.83$ cm	Smallest $D = 12.41 - 8.62$ $= 3.79$ cm

Note that the notation $D \pm \Delta D = 3.81 \pm 0.02$ cm gives the same result.
Check for the sum S following the same steps.

MULTIPLICATION AND DIVISION

Suppose we want to calculate the product P of two measurements $x \pm \Delta x$ and $y \pm \Delta y$. We can write the calculated value as

$$P = x \,.\, y$$

When the errors Δx and Δy happen to reinforce each other, the product P could be as large as

$$\begin{aligned} P + \Delta P &= (x + \Delta x).(y + \Delta y) \\ &= x(1 + \frac{\Delta x}{x}).y(1 + \frac{\Delta y}{y}) \\ &= x.y(1 + \frac{\Delta x}{x})(1 + \frac{\Delta y}{y}) \\ &= x.y(1 + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta x.\Delta y}{x.y}) \end{aligned}$$

The last term $\frac{\Delta x.\Delta y}{x.y}$ in the parenthesis can be neglected since both $\frac{\Delta x}{x}$ and $\frac{\Delta y}{y}$ are small; hence product $\frac{\Delta x.\Delta y}{x.y}$ is much smaller. We have

$$P + \Delta P = x.y(1 + \frac{\Delta x}{x} + \frac{\Delta y}{y})$$

The product P could be as small as

$$P - \Delta P = (x - \Delta x).(y - \Delta y)$$

$$= x(1-\frac{\Delta x}{x}).y(1-\frac{\Delta y}{y})$$

$$= x.y(1-\frac{\Delta x}{x}-\frac{\Delta y}{y}+\frac{\Delta x.\Delta y}{x.y})$$

Again, neglecting the last term in the parenthesis

$$P-\Delta P = x.y(1-\frac{\Delta x}{x}-\frac{\Delta y}{y})$$

Therefore the product P and its maximum possible error ΔP can be written as

$$P \pm \Delta P = x.y \pm x.y(\frac{\Delta x}{x}+\frac{\Delta y}{y})$$

Comparing the two sides of this equation and noting that $P = x.y$, we can write

$$\Delta P = P(\frac{\Delta x}{x}+\frac{\Delta y}{y}) \qquad \text{or} \qquad \frac{\Delta P}{P} = (\frac{\Delta x}{x}+\frac{\Delta y}{y})$$

We see that in the multiplication of two measurements, the maximum relative error $\frac{\Delta P}{P}$ in the product is equal to the sum of the relative errors in the quantities multiplied.

In terms of the percentage errors we can write

$$\frac{\Delta P}{P}\times 100 = \frac{\Delta x}{x}\times 100 + \frac{\Delta y}{y}\times 100$$

Suppose we want to calculate the division D of two measurements $x \pm \Delta x$ and $y \pm \Delta y$. Assume that we have the division $D = \frac{x}{y}$. The division D can be written as

$$D = x.y^{-1}$$

Following the same procedure as for the multiplication we can easily show that

$$\frac{\Delta D}{D}\times 100 = \frac{\Delta x}{x}\times 100 + \frac{\Delta y}{y}\times 100$$

When measured quantities are multiplied or divided, the maximum percentage error of the result is the sum of the percentage errors in the measured quantities used in the calculation.

GN-7. GRAPHS

In order to see the quantitative relationship between the physical quantities involved in an experiment, we express our measurement of these quantities in tabular form. Although we can make some prediction from this *data table,* another very useful way of expressing the data is plotting *a graph* from it. A graph provides a visual picture of the data and from the graph we can deduce the relationship between two related variable physical quantities. In order to plot a graph of two variables, we use rectangular x- and y-axes. The horizontal axis (x-axis *or abscissa)* is used for the *independent variable,* like time t. The vertical axis (y-axis *or ordinate)* is used for the *dependent variable,* like the height h from the ground of a ball falling freely. The location of a point on the graph is defined by its *coordinates* x and y. The coordinates of the point are written as an ordered pair (x, y) and they are measured with respect to the intersection of x- and y-axes called the *origin* (0,0).

A graph should be self-explanatory, and therefore it must have:

1. The titles of the graph on the graph paper (usually written like *Height h versus Time t* or like *Pressure P versus Temperature T*).
2. The student's and, if any, the partner's name and experiment date.
3. Each axis labeled with the quantity plotted and its proper unit.
4. The scaling so that points are distributed as widely as possible over the area of the graph paper used to plot the graph.
5. Simple scales to make calculations straightforward (whole number of squares represent whole unit as a scale).
6. The error bars representing the uncertainty in each reading.

From our measurements, we should have a set of points (x_1, y_1), (x_2, y_2), (x_3, y_3) etc. Where x_1 and y_1 are the coordinates of the first point, and so on. When we plot the data points with their error bars, we do not join all the points together by a zigzag. Instead of this, we try to draw a smooth curve or a straight line, which may not pass through some of the data points. Smooth means that the line we draw does not have to pass exactly through each plotted point, but passes the plotted points as *a curve of best* fit.

The graph we draw, however, indicates how the two quantities depend on each other.

ANALYSIS OF A STRAIGHT LINE GRAPH

Two *linearly related* quantities, x and y, have an algebraic equation of the form

$$y = mx + b$$

where m and b are constants. When the values of such two quantities are plotted, the graph is *a straight line.*

In many experiments, the result of a set of data usually yields a straight-line graph when we plot it. In practice, however, the measured points will be scattered because of the error they contain and we will be required to draw the best straight line passing through the data points.

Picking the best straight line is sometimes difficult, but a transparent ruler may be helpful to make trail lines and try to arrange the line so that an equal number of points lie on either side in a random way. The best straight line, however, must pass inside as many error bars as possible.

Fig.7-1 gives an example of a straight-line graph between two physical quantities x and y. Note that for each measured point the positive and negative maximum possible errors are marked with error bars.

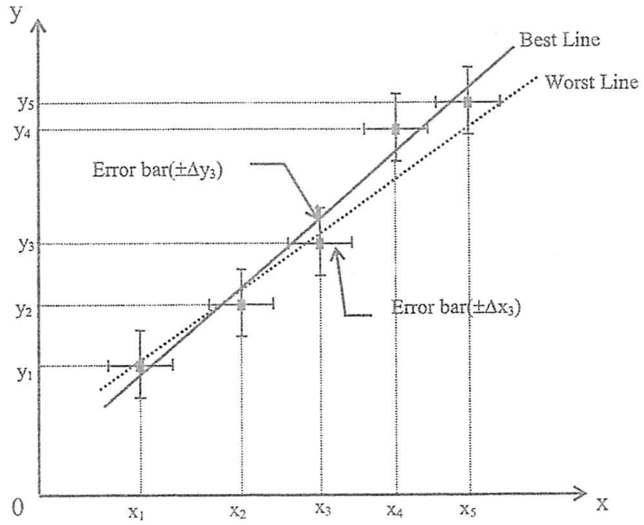

FIGURE 7-1. Example of a straight-line graph with error bars.

After we draw the graph of y versus x we want to determine the two constants m and b in the algebraic equation. The m in the algebraic equation is called the *slope* or *gradient* of the line and it measures the rate at which y changes with x. In order to calculate the slope m, we take two points, (x_1, y_1) and (x_2, y_2), being as far apart as possible on the best straight line fitted to the experimental data points. Slope m is expressed as

$$m = \frac{\Delta y}{\Delta x}$$

where Δy and Δx are the changes in y and x, respectively, as we go from point (x_1, y_1) to point (x_2, y_2). They are given by

$$\Delta y = y_2 - y_1 \qquad \text{and} \qquad \Delta x = x_2 - x_1$$

Thus, slope m can be written as

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note that, the changes $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$ are different from the maximum possible error Δy and Δx in the measurement y and x, respectively. The similar symbolism is merely a coincidence.

The b in the algebraic equation is called the *intercept* and it gives the point where the straight line cuts the y-axis, that is, when $x = 0$, then $y = b$.

The general equation of a straight-line graph passing through the origin (the intercept b=0) is given by $y = mx$. Such a relation indicates a direct proportion between the two quantities x and y; then the slope m is the constant of proportionality.

After plotting the experimental points (x_1, y_1), (x_2, y_2) etc. with corresponding error bars $\pm\Delta x_1$, $\pm\Delta y_1$, etc., and drawing the best straight line through these data points, we determine the slope m and the intercept b from this graph. Actually our task is to determine from the graph the slope as $m \pm \Delta m$ and the intercept as $b \pm \Delta b$, where Δm and Δb are the maximum possible errors in slope m and in intercept b, respectively. These errors can be easily determined by drawing the *worst possible straight line* on the graph. We analyze the worst possible line in the same way to obtain the worst possible slope m' and the worst possible intercept b'; and then we get the possible errors Δm and Δb as

$$\Delta m = |m - m'| \qquad \text{and} \qquad \Delta b = |b - b'|$$

The straight-line graph provides the best way of increasing the accuracy of our experiment. Instead of using a single set of two readings to determine a quantity, we always prefer to collect a set of readings to plot a graph, and we use this graph to obtain the quantity more accurately. As an example, consider a simple electrical experiment in which the resistance R of a wire is determined by measuring the current I flowing through and the corresponding potential difference V across the wire. According to Ohm's law ($V = RI$, the resistance R is related to the measured quantities I and V through the equation

$$R = \frac{V}{I}$$

The error in the calculated R can be reduced by taking several readings of I and V, in each case calculating R and then obtaining an average of all values of R. One disadvantage of this method of determining R is that results differing widely from the average have bigger influence on the calculated value. Therefore, one or two bad results can give a large error in the result and cancel out the value of taking a series of readings and determining the average of them.

If we plot a graph with values of V as the x-axis and values of I as the y-axis then a straight-line graph can be obtained. From the slope of this straight line the resistance R can be found. This procedure is explained later in this part of the laboratory manual.

Example 7- 1:

A body is dropped from a certain height and its speed v at the time t after an arbitrary initial time t = 0 is measured. The following data is obtained. Analyze the data by plotting a graph v versus t.

Speed, v (m/s)	Time, t (sec)
1.05 ± 0.08	0.025 ± 0.001
1.20 ± 0.08	0.050 ± 0.001
1.40 ± 0.08	0.075 ± 0.001
1.62 ± 0.08	0.100 ± 0.001

Table 7- 1. The speed of a falling body.

Solution:

Let's plot a graph of v versus t using the data in Fig.7-2. In doing so, we put v values on the y-axis and t values on the x-axis. Note, how the title is put with the date and names; how the axes are labeled with appropriate units; how only a few principal values are marked on each axis; how each experimental point is denoted with error bars. The error is appreciable in v values only and the error in t values is neglected. The best straight line and the worst possible line are drawn. On each line we take two points being as far apart as possible.

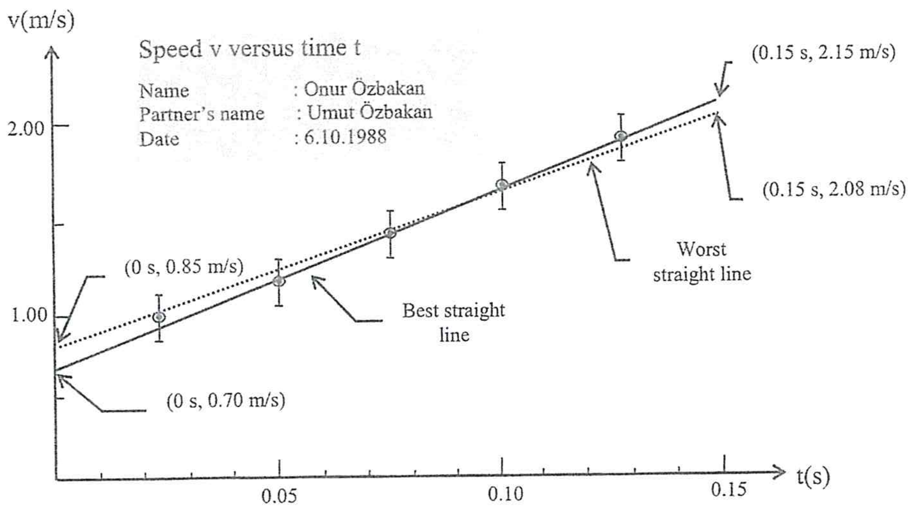

FIGURE 7-2. The speed v versus time t graph and analysis of a straight line graph.

Using the end points of the best straight-line slope, m can be determined. The two end points are (0 s, 0.70 m/s) and (0.15 s, 2.15 m/s) Slope m of the best straight line is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.15\ m/s\ -\ 0.85\ m/s}{0.15\ s\ -\ 0\ s} = 9.67\ m/s^2$$

Similarly, for the worst possible line using the two end points (0 s, 0.85 m/s) and (0.15 s, 2.08 m/s) the slope m' is given by

$$m' = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.15\ m/s\ -\ 0.85\ m/s}{0.15\ s\ -\ 0\ s} = 9.67\ m/s^2$$

The maximum possible error in the slope is

$$\Delta m = |m - m'| = |9.67 - 8.20| = 1.47\ m/s^2$$

The slope of the best straight line with its maximum possible error is

$$m \pm \Delta m = 9.67 \pm 1.47\ m/s^2$$

We can round off the result to the first column in which a nonzero error occurs. Thus we can write the slope as

$$m \pm \Delta m = 10 \pm 1\ m/s^2$$

The intercept b of the best straight line and b' of the worst possible line from the graph are

$$b = 0.70\ m/s$$
$$b' = 0.85\ m/s$$

The maximum possible error in the intercept is

$$\Delta b = |b - b'| = |0.70 - 0.85| = 0.15\ m/s$$

The intercept of the best straight line with its maximum possible error is

$$b \pm \Delta b = 0.70 \pm 0.15\ m/s$$

We can round off the result to the first column in which a nonzero error occurs. Thus we can write the intercept as

$$b \pm \Delta b = 0.7 \pm 0.2\ m/s$$

Note that this method is the simplest and easiest way to analyze a straight-line graph. The *method of least squares* to draw the best straight line through a set of experimental data is out of the scope of this manual.

One common example of motion with almost constant acceleration is that of an object falling toward the earth from a certain height. When the air resistance is neglected we find that all objects, whatever their size, mass, or composition, fall with the same acceleration, and if the height they fall from is not too great, the acceleration remains constant throughout the motion. Such a motion, in the absence of air friction, is called *free fall*.

The acceleration of a freely falling body is called the *acceleration due to gravity* and it is directed down toward the center of the earth. It is denoted by the symbol g and near the earth's surface its magnitude is approximately 9.8 m/s^2 (= 980 cm/s^2). However, the exact value of it changes with latitude and altitude.

The equation relating the speed v and time t (measured from an arbitrary initial time t=0) of an object falling freely down to the ground from a certain height is

$$v = v_o + gt$$

where v_0 is the initial speed and g is the acceleration due to gravity. Let's write this equation as $v = gt + v_0$ and compare it with the general equation of a straight-line $y = mx + n$, written in terms of two variables x and y.

To do so, let's put the two equations one after another comparing the corresponding terms by the use of arrows in the following fashion

$$y = mx + b$$
$$v = gt + v_0$$

As seen from the correspondence between the two equations, the equation $v = gt + v_0$ is a straight-line equation. From the graphical analysis of v versus t graph we have slope m being equal to acceleration g due to gravity and intercept b to the initial speed v_0. From our graphical analysis of the experimental data, we obtain the physical quantities g and v_0 as

$$g \pm \Delta g = 10 \pm 1\ m/s^2 \quad \text{and} \quad v_0 \pm \Delta v_0 = 0.7 \pm 0.2\ m/s$$

NON-LINEAR GRAPHS

The following are some non-linear relations in physics:

INVERSE PROPORTION

A graph of pressure P versus volume V (Fig.7-3a) for an ideal gas at constant temperature yields the relationship

$$PV = k$$

where k is a constant. Note that, a graph of P versus $\frac{1}{V}$ (Fig.7-3b) would be a straight line through the origin since the relationship $P = k\,(\frac{1}{V})$ is equivalent to the general equation of a straight-line graph passing through the origin.